The Higgs Mechanism and Loop-induced Decays of a Scalar into Two Z Bosons

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Abstract

We discuss general on-shell couplings of a scalar with two Z bosons using an operator analysis. In addition to the operator originated from the Higgs mechanism, two dimension-five operators, one CP-even and one CP-odd, are generated only at the loop-level. Simple formulas are derived for the differential decay distributions when the Z pair subsequently decay into four leptons by computing the helicity amplitudes, from which it is shown the CP-odd operator merely induces a phase shift in the azimuthal angular distribution between the two decay planes of the Z bosons. We also investigate new physics scenarios giving rise to loop-induced decays of a scalar into ZZ pair, and argue that the total decay width of such a scalar would be order-of-magnitude smaller than that of a Higgs boson, should such decays be observed in the early running of the LHC. Therefore, the total decay width alone is a strong indicator of the Higgs nature, or the lack thereof, of a scalar resonance in ZZ final states. In addition, we study the possibility of using the azimuthal angular distribution to disentangle effects among all three operators.

I. INTRODUCTION

In the standard model (SM), the electroweak gauge bosons obtain their masses through the Higgs mechanism, which postulates the existence of a scalar particle whose vacuum expectation value (VEV) breaks the electroweak $SU(2)_L \times U(1)_Y$ symmetry down to $U(1)_{em}$. If the scalar, the Higgs boson, is a $SU(2)_L$ doublet denoted by $H = (h^+, h)^T$, then its kinetic term

$$|D_\mu H|^2 = \left| \left(\partial_\mu - ig\frac{\sigma^a}{2} W^a_\mu - ig'\frac{1}{2} B_\mu \right) H \right|^2 \tag{1}$$

contains mass terms for electroweak gauge bosons after the neutral component of the Higgs doublet gets a VEV, $\langle H \rangle = (0, v)^T/\sqrt{2}$, where g and g' are the gauge couplings for the $SU(2)_L$ and $U(1)_Y$, respectively, and σ^a are the Pauli matrices. Using the mass eigenbasis

$$W^\pm_\mu = \frac{1}{\sqrt{2}}(W^1_\mu \pm iW^2_\mu), \quad Z_\mu = \frac{gW^3_\mu - g'B_\mu}{\sqrt{g^2 + g'^2}}, \quad A_\mu = \frac{g'W^3_\mu + gB_\mu}{\sqrt{g^2 + g'^2}}, \tag{2}$$

one finds from Eq. (1) the following masses

$$m_W = \frac{1}{2} g v, \qquad m_Z = \frac{1}{2}\sqrt{g^2 + g'^2}\, v, \qquad m_A = 0. \tag{3}$$

Furthermore, there are also three-point and four-point couplings from the Higgs kinetic term derived by replacing $m_V \to m_V(1 + h/v)$ in the gauge boson mass term:

$$\left(1 + \frac{h}{v}\right)^2 m_V^2 V_\mu V^\mu, \tag{4}$$

where $V = W, Z$. The form of the hVV coupling is completely determined by the electroweak gauge invariance to be

$$-2i\frac{m_V^2}{v} g_{\mu\nu}. \tag{5}$$

Therefore, measurements of the three-point vertex in Eq. (5) will be a striking confirmation of the Higgs mechanism.

Experimentally, the hVV vertex plays an important role in discovering the Higgs boson at the Large Hadron Collider (LHC). For a Higgs mass above 150 GeV or so, the branching ratio is dominated by decays into WW and ZZ [1]. In particular, $h \to ZZ \to 4\ell$ is the gold-plated mode for the discovery of a moderately heavy ($\gtrsim$ 180 GeV) Higgs boson, which is a very clean signature with relatively small backgrounds. The excellent energy resolution of the reconstructed electrons and muons leads to a clear 4-lepton invariant mass peak, which allows for precise measurements of the mass and width of the Higgs boson [2].

Given that so far all data from collider experiments agree with predictions of the SM quite well, there are very few experimental hints on what could (and could not) be seen at the LHC. Therefore, if a new scalar resonance is observed in the WW and ZZ final states, it is perhaps prudent to proceed without presuming the discovery of a Higgs boson whose VEV gives masses to the W and Z bosons. Only until after one could verify the decay indeed occurs through the three-point coupling in Eq. (5), can one gain some confidence in the Higgs mechanism as the origin of electroweak symmetry breaking (EWSB).

In this work we study the physics giving rise to decays of a scalar into two Z bosons, with an emphasis on probing the Higgs nature of the scalar. Such a final state is interesting in its own right because of the high degree of symmetry in two identical spin-1 particles. Early studies of such systems resulted in the Landau-Yang theorem, which forbids decays of a spin-1 particle into two photons [3]. Recently similar arguments to the Landau-Yang theorem have been extended to decays of a massive spin-1 particle, the Z' boson, into two Z bosons [4]. There it was discovered that the azimuthal angle between the two decay planes of the Z is a very useful observable in discerning different interactions of the Z' with the Z bosons.

Here we consider the production of a scalar S in the gluon fusion channel, which is the dominant production mechanism of a Higgs boson at the LHC [1], and its subsequent decays into two Z bosons. We do not assume the scalar S plays the role of the Higgs boson in the Higgs mechanism. In particular, we point out non-Higgs-like couplings are induced only at the loop-level, and investigate in detail implications on the underlying new physics. Since we presume the scalar S and the Z bosons are all produced on-shell, implying $m_S \geq 2m_Z$, our analysis is different and complimentary to studies on anomalous Higgs couplings in the vector boson fusion production, where the vector bosons are off-shell [5, 6].[1] (Measurements of anomalous Higgs couplings at the linear collider have been studied in [7].) Differential distributions of a scalar decaying into $ZZ \to 4\ell$ in the general case have been computed in Refs. [8, 9]. However, applying the symmetry argument as in Refs. [3, 4] would allow us to simplify the decay distributions dramatically, making it transparent the usefulness of the aforementioned azimuthal angle. We also argue that the total width of a scalar decaying to

[1] It is worth pointing out that at the LHC the production rate in the gluon fusion channel is an order of magnitude larger than the vector fusion production through out a wide range of Higgs mass.

ZZ through loop-induced effects should be much smaller than that of a Higgs-like scalar, if the loop-induced decays should be observed at the LHC in the early running. Therefore measurements on the total width alone is a smoking gun signal for the Higgs nature of the scalar resonance.

This paper is organized as follows: in the next section we compute the differential distribution of the decay of a scalar into $ZZ \to 4\ell$ using the helicity amplitudes method, followed by a discussion on the possible new physics giving rise to loop-induced couplings. In Section IV we perform simulations on the total decay width measurements as well as azimuthal angular distributions between the two decay planes of the Z boson. Then we conclude in Section V. We also provide two appendices, one on a toy model in which the loop-induced coupling is mediated by the heavy W'-boson loop and the other on the Lorentz-invariant construction of the aforementioned azimuthal angle.

II. HELICITY AMPLITUDES FOR $S \to Z(\lambda_1, k_1) Z(\lambda_2, k_2) \to (\ell_1 \bar{\ell}_1)(\ell_2 \bar{\ell}_2)$

We use the notation $(\lambda_i, k_i), i = 1, 2$, to denote the helicity state and momentum of the two Z bosons in the laboratory frame. Assuming all three particles are on-shell, the possible helicity states $\Psi^{\lambda_1\lambda_2}$ of the Z pair are determined by conservation of angular momentum to be Ψ^{++}, Ψ^{--}, and Ψ^{00}, from which we see the parity-even combinations are $\Psi^{++} + \Psi^{--}$ and Ψ^{00} while the parity-odd one is $\Psi^{++} - \Psi^{--}$. In terms of effective Lagrangian, the three helicity amplitudes are described by the following three operators

$$\mathcal{L}_{eff} = \frac{1}{2} m_S \, S \left(c_1 Z^\nu Z_\nu + \frac{1}{2} \frac{c_2}{m_S^2} Z^{\mu\nu} Z_{\mu\nu} + \frac{1}{4} \frac{c_3}{m_S^2} \epsilon_{\mu\nu\rho\sigma} Z^{\mu\nu} Z^{\rho\sigma} \right), \tag{6}$$

where $Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu$ is the field strength, and $c_i, i = 1, 2, 3$, are dimensionless constants. A fourth operator, $Z_{\mu\nu}(Z^\mu \partial^\nu S - Z^\nu \partial^\mu S)$, is related to the c_1 and c_2 terms upon the equation of motion. The tensor structure of the decay amplitude of $S \to Z_1(k_1^\alpha) + Z_2(k_2^\beta)$ is

$$\epsilon_1^\alpha \epsilon_2^\beta \mathcal{M}_{\alpha\beta} = m_S \; \epsilon_1^\alpha \epsilon_2^\beta \left\{ c_1 \, g_{\alpha\beta} - \frac{c_2}{m_S^2} \left[g_{\alpha\beta}(k_1 \cdot k_2) - (k_1)_\alpha (k_2)_\beta \right] + \frac{c_3}{m_S^2} \epsilon_{\alpha\beta\gamma\delta} k_1^\gamma k_2^\delta \right\}, \tag{7}$$

where ϵ_1^α and ϵ_2^β are the polarization tensors of Z_1 and Z_2, respectively. Terms in Eq. (7) proportional to c_2 and c_3 are the so-called anomalous Higgs couplings.

Following the method and convention in Ref. [4], we calculate the helicity amplitudes

$\mathcal{M}_{\lambda_1\lambda_2}$:

$$\mathcal{M}_{\pm\pm} = \frac{m_S}{2}\left[2c_1 - c_2\left(1-\frac{2m_Z^2}{m_S^2}\right) \pm ic_3\sqrt{1-\frac{4m_Z^2}{m_S^2}}\ \right], \tag{8}$$

$$\mathcal{M}_{00} = m_S\left[c_1\left(1-\frac{m_S^2}{2m_Z^2}\right)+c_2\frac{m_Z^2}{m_S^2}\right]. \tag{9}$$

Notice that $\mathcal{M}_{00}$ is real while the amplitudes $\mathcal{M}_{\pm\pm}$ are complex in the presence non-zero c_3. Therefore, we can parametrize the three helicity amplitudes in terms of two real numbers, $\mathcal{M}_T$ and $\mathcal{M}_L$, and one phase δ:

$$\mathcal{M}_{++} = M_T e^{i\delta}\ , \quad \mathcal{M}_{--} = M_T e^{-i\delta}\ , \quad \mathcal{M}_{00} = M_L\ , \tag{10}$$

where

$$M_T = \frac{m_S}{2}\left\{\left[2c_1 - c_2\left(1-\frac{2m_Z^2}{m_S^2}\right)\right]^2 + c_3^2\left(1-\frac{4m_Z^2}{m_S^2}\right)\right\}^{1/2}, \tag{11}$$

$$\delta = \arctan\frac{c_3(1-4m_Z^2/m_S^2)^{1/2}}{2c_1 - c_2(1-2m_Z^2/m_S^2)}\ . \tag{12}$$

When the two Z bosons further decay into $(\ell_1\bar{\ell}_1)(\ell_2\bar{\ell}_2)$, the phase δ enters into the differential distribution in a simple way. To see this, recall that the angular distribution of the decay $Z_i \to \ell_i\bar{\ell}_i$ in the rest frame of Z_i has the dependence $e^{im_i\phi_i}$, where $m_i = 0, \pm 1$ is the spin projection along the z axis and ϕ_i is the azimuthal angle. Since only the relative angle ϕ is physical we set $\phi_1 = 0$ and $\phi_2 = \phi$. (See Fig. 1.) From Eq. (10) we see δ only enters as a phase shift in $\phi \to \phi + \delta$. Furthermore, the angular dependence of the differential decay rate is schematically

$$\frac{d\Gamma}{\Gamma d\phi} \sim |a_1 + a_2 e^{i(\phi+\delta)} + a_3 e^{-i(\phi+\delta)}|^2 \sim b_1 + b_2\cos(\phi+\delta) + b_3\cos(2\phi+2\delta). \tag{13}$$

For a Z' boson decaying into the ZZ pair, the $\cos 2\phi$ term is absent in Eq. (13) and a similar phase shift δ' enters as $\phi \to \phi + 2\delta'$ [4].

Using g_L and g_R to denote the coupling of the Z boson to the left-handed and right-handed leptons, respectively, we arrive at the differential distribution of $S \to Z_1 Z_2 \to (\ell_1\bar{\ell}_1)(\ell_2\bar{\ell}_2)$

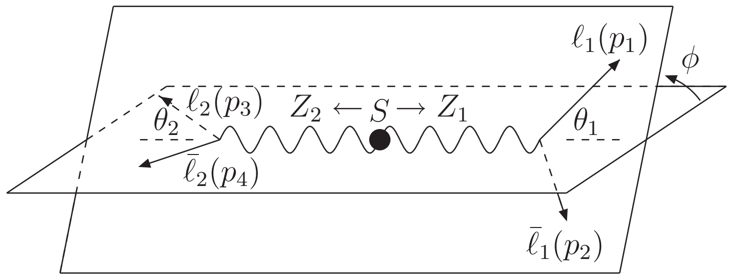

FIG. 1: *Two decay planes of $Z_1 \to \ell_1\bar{\ell}_1$ and $Z_2 \to \ell_2\bar{\ell}_2$ define the azimuthal angle $\phi \in [0, 2\pi]$ which rotates ℓ_2 to ℓ_1 in the transverse view. The polar angles θ_1 and θ_2 shown are defined in the rest frame of Z_1 and Z_2, respectively.*

following the method of helicity amplitudes [10]:

$$\frac{d\Gamma}{\Gamma d\cos\theta_1 d\cos\theta_2 d\phi} = \frac{1}{N}\left\{\frac{1}{2}\sin^2\theta_1\sin^2\theta_2\cos(2\phi+2\delta)+\right.$$
$$\frac{M_L}{M_T}\left[\frac{1}{2}\sin 2\theta_1\sin 2\theta_2 + 2\left(\frac{g_R^2-g_L^2}{g_R^2+g_L^2}\right)^2\sin\theta_1\sin\theta_2\right]\cos(\phi+\delta) + \frac{M_L^2}{M_T^2}\sin^2\theta_1\sin^2\theta_2$$
$$\left.+\frac{1}{2}(1+\cos^2\theta_1)(1+\cos^2\theta_2) + 2\left(\frac{g_R^2-g_L^2}{g_R^2+g_L^2}\right)^2\cos\theta_1\cos\theta_2\right\}, \tag{14}$$

where the definition of θ_1, θ_2, and ϕ are given in Fig. 1. Integrating over the polar angles, we get the expression

$$\frac{d\Gamma}{\Gamma d\phi} = \frac{1}{N}\left\{\frac{8}{9}\cos(2\phi+2\delta)\right.$$
$$\left.+\frac{\pi^2}{2}\frac{M_L}{M_T}\left(\frac{g_R^2-g_L^2}{g_R^2+g_L^2}\right)^2\cos(\phi+\delta) + \frac{16}{9}\left(\frac{M_L^2}{M_T^2}+2\right)\right\}. \tag{15}$$

The normalization factor is given by integrating the above expression,

$$N = \frac{32\pi}{9}\left(\frac{M_L^2}{M_T^2}+2\right) . \tag{16}$$

Let's consider turning on c_i one at a time:

- $c_1 \neq 0$ and $c_2 = c_3 = 0$:

$$\left.\frac{M_L}{M_T}\right|_{c_1\neq 0} = 1 - \frac{m_S^2}{2m_Z^2} \qquad \text{and} \qquad \delta = 0 . \tag{17}$$

This is the case when S plays the role of the Higgs boson in the Higgs mechanism. Since we assume $m_S \geq 2m_Z$ for on-shell production, we see $|M_L/M_T|_{c_1\neq 0} \geq 1$ and

the longitudinal component of the Z dominates over the transverse components in the decay, especially in the limit of large m_S.

- $c_2 \neq 0$ and $c_1 = c_3 = 0$:

$$\left.\frac{M_L}{M_T}\right|_{c_2\neq 0} = \frac{-1}{1-m_S^2/(2m_Z^2)} = -\left(\left.\frac{M_L}{M_T}\right|_{c_1\neq 0}\right)^{-1} \qquad \text{and} \qquad \delta = 0 \; . \tag{18}$$

  In this case $|M_L/M_T|_{c_2\neq 0} < 1$ and the transverse polarization of the Z dominates in the decay.

- $c_3 \neq 0$ and $c_1 = c_2 = 0$:

$$\left.\frac{M_L}{M_T}\right|_{c_3\neq 0} = 0 \qquad \text{and} \qquad \delta = \frac{\pi}{2} \; . \tag{19}$$

  This is a particularly simple case, as the normalized differential distribution in Eq. (15) reduces to

$$\frac{d\Gamma}{\Gamma d\phi} = \frac{1}{2\pi}\left(1 - \frac{1}{4}\cos 2\phi\right) \; . \tag{20}$$

Previous analysis assuming the SM Higgs boson can be found in Ref. [11], while Refs. [12, 13] addressed the CP violation due to the simultaneous presence of c_1 and c_3 terms. Our general and simple result in Eqs. (14) and (15) is in agreement with the lengthy expressions in Ref. [9]. Furthermore, our analysis makes it clear that the effect of a non-zero c_3, which is CP-odd, is to induce a phase shift in the azimuthal angular distribution.

III. NEW PHYSICS AND LOOP-INDUCED DECAYS OF S

Among the three operators in Eq. (6), c_1 has the form of the three-point coupling in the Higgs mechanism and could be present at the tree-level,[2] while both c_2 and c_3 are higher dimensional operators induced only at the loop level [14]. If $c_1 = 0$ at the tree-level, the scalar S is not responsible for giving W and Z bosons a mass. We also assume the existence of the following gluonic operators,

$$\frac{c_{g2}}{4m_S} S G^a_{\mu\nu} G^{a\,\mu\nu} \qquad \text{and} \qquad \frac{c_{g3}}{8m_S} S \epsilon^{\mu\nu\rho\sigma} G^a_{\mu\nu} G^a_{\rho\sigma}, \tag{21}$$

[2] c_1 could also be generated through dimension-five operators such as $S|D_\mu H|^2$, which is suppressed by a high mass scale comparing to Eq. (5). We will not consider this possibility further in this work.

so as to allow for the production of S in the gluon fusion channel. In the SM it is well-known that c_{g2} is induced by the top-quark triangle loop when S is the Higgs boson. In fact, c_2 is also present in the SM through the W-boson as well as the top-quark loop [15], which is nonetheless overwhelmed by the tree-level c_1 given in Eq. (5). On the other hand, the CP-odd operators c_3 and c_{g3} can be generated by a fermion triangle loop when the fermion has an axial coupling with the scalar S [12].

At the LHC, the event rate $B\sigma(gg \to S \to ZZ)$ is

$$B\sigma(gg \to S \to ZZ) = \sigma(gg \to S) \times \mathrm{Br}(S \to ZZ) = \sigma(gg \to S) \times \frac{\Gamma(S \to ZZ)}{\Gamma_{\mathrm{total}}} \ , \quad (22)$$

where the total decay width Γ_{total} is given by summing over all decay channels, including possible decays into SM fermion pair $\bar{f}f$,

$$\Gamma_{\mathrm{total}} = \sum_{V=g,W,Z,\gamma} \Gamma(S \to VV) + \sum_f \Gamma(S \to \bar{f}f) \ . \quad (23)$$

A few model-independent observations are in order:

- While the decay channel into fermions may or may not exist, electroweak symmetry ensures the existence of decay channels into WW and $\gamma\gamma$ once $S \to ZZ$ is observed. Establishing the production $gg \to S$ also guarantees a decay channel into two gluons.

- If the event rate $B\sigma$ is comparable to the SM expectation of a Higgs boson, then the branching ratio into ZZ pair should be sizable

  $$\mathrm{Br}(S \to ZZ) \gtrsim \mathcal{O}(10^{-1}) \ . \quad (24)$$

  The SM Higgs production and decay $gg \to h \to ZZ \to 4\ell$ has an event rate in the order of 5 fb after multiplying $\sigma \times \mathrm{Br}$ with the pre-selection efficiency [2]. Therefore if $\sigma \times \mathrm{Br}$ is an order-of-magnitude smaller that that of a SM Higgs, it would require an integrated luminosity of 300 fb^{-1} to achieve 5σ significance for discovery, which is clearly beyond the early running of the LHC.

- If $S \to ZZ$ is observed to occur through the loop-induced operators in the early LHC data, then a sizable $\mathrm{Br}(S \to ZZ)$ implies the total decay width

  $$\Gamma_{\mathrm{tot}} = \frac{\Gamma(S \to ZZ)}{\mathrm{Br}(S \to ZZ)} \quad (25)$$

  should also be one-loop suppressed.

- Similarly, loop-induced $S \to ZZ$ and a sizable $\mathrm{Br}(S \to ZZ)$ imply[3]

$$\frac{\Gamma(S \to ZZ)}{\Gamma(S \to gg)} \sim \mathcal{O}\left(\frac{c_i^2}{c_{gi}^2}\right) \gtrsim \mathcal{O}(10^{-1}) \ . \tag{26}$$

  Since we expect c_i and c_{gi} to be proportional to the electroweak coupling $\alpha_{\rm ew}$ and the strong coupling α_s, respectively, a large multiplicity factor ($\gtrsim \mathcal{O}(1)$) in c_i should be present.

In the following we investigate new physics scenarios where the production and decay into ZZ of S occur predominantly through loop-induced operators. Such possibilities arise naturally if S is a SM singlet and couples to SM matter only through a messenger sector. In particular we focus on cases with a sizable branching ratio $\mathrm{Br}(S \to ZZ)$ as in Eq. (24), so that S would have a comparable event rate to that of a SM Higgs boson.

A. Fermion Loop-induced $S \to gg$

In the SM gluon fusion production is induced by the top quark loop [1],

$$c_{g2}^{(SM)} = \frac{\sqrt{2}\alpha_s}{3\pi}\frac{m_S}{v} \ . \tag{27}$$

It is well-known that this coefficient is related to the top contribution to the gluon two-point function from the Higgs low-energy theorem [16]. If the messenger sector contains a pair of heavy vector-like fermions (Q^c, Q) in the fundamental representation of $SU(3)_c$ with the interaction

$$m_Q Q^c Q + y_Q S\, Q^c Q, \tag{28}$$

then its contribution to the gluon two-point function is

$$-\frac{1}{4}\left[1 - \frac{g_s^2}{16\pi^2} b_F^{(3)} \log\frac{M_Q^2(\mathcal{S})}{\mu^2}\right] G^a_{\mu\nu} G^{a\,\mu\nu}, \tag{29}$$

where $b_F^{(3)} = 2/3$ is the contribution to the one-loop beta function of QCD from a Dirac fermion and $M_Q(\mathcal{S}) = m_Q + y_Q \mathcal{S}$ is the mass of the new heavy fermion Q when turning

[3] In Eq. (26) we have neglected an extra factor for decaying into massive gauge bosons, which is order unity unless m_S is very close to the $2m_Z$ threshold.

on the scalar as a background field $S \to \mathcal{S}$. To obtain the scalar-gluon-gluon coupling, the Higgs low-energy theorem instructs us to expand Eq. (29) to the first order in $\mathcal{S}$ [17]:

$$c_{g2} = \frac{\alpha_s}{3\pi} \frac{m_S}{m_Q} y_Q. \tag{30}$$

Strictly speaking, the low-energy theorem applies only when the mass of the particle in the loop is much larger than the scalar mass, $m_S^2/(4m_Q^2) \ll 1$, so that the loop diagram can be approximated by a dimension-five operator. We will always work in this limit in the present study. The partial width of $S \to gg$ can be computed:

$$\Gamma(S \to gg) = \frac{1}{8\pi} c_{g2}^2 \, m_S = \frac{\alpha_s^2}{72\pi^3} \frac{m_S^3}{m_Q^2} y_Q^2. \tag{31}$$

B. Fermion Loop-induced $S \to ZZ$

Next we consider the case when the messenger sector contains vector-like fermions (L^c, L) charged under the electroweak gauge group with the interaction

$$m_L L^c L + y_L S \, L^c L, \tag{32}$$

where (L^c, L) are in the fundamental representation of $SU(2)_L$ and carry the hypercharge Y_L under $U(1)_Y$. The contribution of L to the two-point function of the Z boson is simply

$$-\frac{1}{4}\left[1 - \frac{e^2 N_c}{16\pi^2 c_w^2 s_w^2}\left(c_w^4 b_F^{(2)} + s_w^4 b_F^{(1)} Y_L^2 d_F^{(2)}\right) \log \frac{M_L^2(\mathcal{S})}{\mu^2}\right] Z_{\mu\nu} Z^{\mu\nu} \ , \tag{33}$$

where $b_F^{(2)} = 2/3$, $b_F^{(1)} = 4/3$, N_c is the dimensionality of the $SU(3)_c$ representation L belongs to, and $d_F^{(2)} = 2$ is the dimensionality of the $SU(2)$ fundamental representation. In addition, c_w and s_w are the cosine and sine of the Weinberg angle. Then we compute

$$c_2^L \ = \ \frac{\alpha_{em}}{3\pi} \frac{N_c}{c_w^2 s_w^2}\left(c_w^4 + 2 s_w^4 Y_L^2 d_F^{(2)}\right) \frac{m_S}{m_L} y_L \ , \tag{34}$$

$$\Gamma^{(f)}(S \to ZZ) \ = \ \mathcal{P}\left(\frac{m_Z^2}{m_S^2}\right) \frac{1}{64\pi} (c_2^L)^2 m_S \tag{35}$$

$$= \ \mathcal{P}\left(\frac{m_Z^2}{m_S^2}\right) \frac{\alpha_{em}^2}{576\pi^3} \frac{N_c^2}{c_w^4 s_w^4}\left(c_w^4 + 2 s_w^4 Y_L^2 d_F^{(2)}\right)^2 \frac{m_S^3}{m_L^2} y_L^2 \ , \tag{36}$$

where $\mathcal{P}(x) = \sqrt{1-4x}\,(1 - 4x + 6x^2)$ is a factor correcting for the massive final states in the decay width. Notice the additional difference between Eq. (31) and Eq. (35) due to a color factor of 8 since there are eight gluons in the final states for $S \to gg$.

It is worth commenting that, since $SU(2)_L$ is broken and gauge invariance does not forbid a mass term for the Z boson, one might expect a contribution to c_1 be generated at the one-loop level. However, recall that vector-like fermions do not give one-loop corrections to the Z boson mass term, and thus make no contributions to the $m_Z^2 Z_\mu Z^\mu$ operator which would have given a contribution to c_1 after applying the Higgs low-energy theorem. This argument suggests that any contribution to c_1 at one-loop level would come from applying the Higgs low-energy theorem to operators with four-derivatives such as $(\Box Z_\mu)^2$, which upon using the equation of motion is suppressed by $(m_Z/m_L)^4$ and can be safely neglected for a heavy m_L. We explicitly computed the fermion triangle loop diagram in a large mass expansion, $m_L \to \infty$, and verified that the first contribution to c_1 indeed starts at $(m_Z/m_L)^4$.

C. Gauge Boson Loop-induced $S \to ZZ$

The last possibility we consider is when the messenger sector includes a new set of heavy electroweak gauge bosons (W', Z'). In the SM the W contribution to the loop-induced decay of the Higgs into $\gamma\gamma$ dominates over the one from the top-quark loop due to a large beta function coefficient "7" multiplying the W loop result [16]. One may expect a similar situation for the W' contribution to the loop-induced decay into ZZ. Given the existence of two sets of electroweak gauge bosons, the simplest model must contain two copies of gauged $SU(2)$. Schematically, the symmetry breaking pattern is a two-step process:

$$SU(2)_1 \times SU(2)_2 \times U(1)_Y \to SU(2)_L \times U(1)_Y \to U(1)_{em} \ , \tag{37}$$

where the two $SU(2)$'s are broken down to the vectorial subgroup, identified with $SU(2)_L$, at a high scale f_1 using a linear sigma model. Subsequently $SU(2)_L \times U(1)_Y$ is broken down to $U(1)_{em}$ at a low scale $f_2 = v$ following the Higgsless model [18].

In the Appendix A we explicitly construct a toy model whose gauge sector is the same as the three-site Higgsless model [19–21], although we are interested in a different corner of parameter space, $\epsilon \equiv (f_2/f_1)^2 \ll 1$. For example, if $f_1 \approx 1$ TeV and $f_2 = v \approx 246$ GeV, we have $\epsilon \approx 0.06 \ll 1$. In this case the W' and Z' can be as light as several hundreds GeV for weakly coupled theories. We computed the W' contribution to the ZZ self-energy. At leading order in ϵ,

$$-\frac{1}{4}\left[1 - \frac{e^2}{16\pi^2 c_w^2 s_w^2}(7c_w^4)\log\frac{M_{W'}^2(\mathcal{S})}{\mu^2}\right] Z_{\mu\nu}Z^{\mu\nu} \ , \tag{38}$$

where we see the same large coefficient as in the scalar coupling to two photons. In the set up we have in the Appendix A, $M_{W'}^2(\mathcal{S}) = \frac{1}{2}(g_1^2 + g_2^2)(f_1 + \mathcal{S})^2$ at leading order in ϵ, which leads to

$$c_2^{W'} = \frac{7\alpha_{em}}{2\pi}\frac{c_w^2}{s_w^2}\frac{m_S}{f_1} \ . \tag{39}$$

Using Eq. (35) we arrive at

$$\Gamma^{(W')}(S \to ZZ) = \mathcal{P}\left(\frac{m_Z^2}{m_S^2}\right)\frac{49\alpha_{em}^2}{256\pi^3}\frac{c_w^4}{s_w^4}\frac{m_S^3}{f_1^2} \tag{40}$$

There is also a contribution to c_1 induced at one-loop by the W' boson that is suppressed by ϵ, which we ignore.

Given Eqs. (31), (35), and (40), we can now compare $\Gamma(S \to gg)$ with $\Gamma(S \to ZZ)$ and see that the decay width into ZZ can easily be comparable to the decay width into two gluons. This is especially the case for the W' loop due to the large coefficient in Eq. (39):

$$\frac{\Gamma^{(W')}(S \to ZZ)}{\Gamma(S \to gg)} \sim 0.75 \times \mathcal{P}\left(\frac{m_Z^2}{m_S^2}\right)\frac{m_Q^2}{f_1^2 y_Q^2} \ . \tag{41}$$

Even in the case of purely fermionic contribution in $S \to ZZ$, assuming the fermion (L^c, L) carries no hypercharge, the ratio of the two partial widths is

$$\frac{\Gamma^{(f)}(S \to ZZ)}{\Gamma(S \to gg)} \sim 0.01 N_c^2 \times \mathcal{P}\left(\frac{m_Z^2}{m_S^2}\right)\frac{m_Q^2 y_L^2}{m_L^2 y_Q^2} \ , \tag{42}$$

which could still be $\mathcal{O}(0.1)$ if the multiplicity factor $N_c \gtrsim 3$. This could be achieved if the fermion (L^c, L) is also in the fundamental representation of $SU(3)_c$, resulting in $N_c = 3$. In the end, we have demonstrated that new physics scenarios giving rise to loop-induced decays of S into gauge bosons could easily give a significant branching ratio into ZZ bosons.

IV. OBSERVABLES AND SIMULATIONS

In this section we discuss two observables which could be useful in disentangling whether the scalar coupling to Z bosons is as predicted by the Higgs mechanism or induced by new physics at the loop-level.

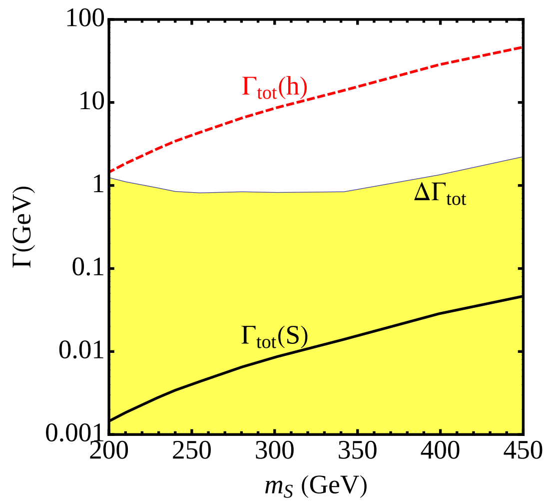

FIG. 2: *The dashed line is the total decay width for a SM Higgs boson and the solid line is that of a scalar S whose width is three orders of magnitude smaller. The yellow (shaded) region is the detector resolution.*

A. The Line Shape

Among the three possible on-shell couplings of S with Z bosons, only c_1 could be present at the tree-level with an order unity coupling when S plays the role of the Higgs boson in the Higgs mechanism, while both c_2 and c_3 are non-zero only at one-loop level. This observation suggests that the total width of a scalar decaying through c_2 and c_3 must be much smaller than that of a scalar decaying through c_1, in order for the decay channel to be observable in the early LHC running. Using the SM Higgs as an example, the partial decay width $\Gamma(h \to VV)$ is

$$\delta_V \mathcal{P}\left(\frac{m_V^2}{m_h^2}\right) \frac{G_F m_h^3}{16\sqrt{2}\pi} , \qquad V = W, Z , \tag{43}$$

where G_F is the Fermi constant and $\delta_W = 2\delta_Z = 2$. Comparing with the partial decay width of a W'-loop induced decay, we see

$$\frac{\Gamma^{(W')}(S \to ZZ)}{\Gamma(h \to ZZ)} \sim 10^{-3} , \tag{44}$$

for a W' mass as light as 500 GeV. As emphasized previously, in order for the event $gg \to S \to ZZ$ to be observable at the LHC with say 30 fb^{-1} luminosity, the branching ratio $\text{Br}(S \to ZZ)$ should be sizable and comparable to that of a SM Higgs into ZZ. It then

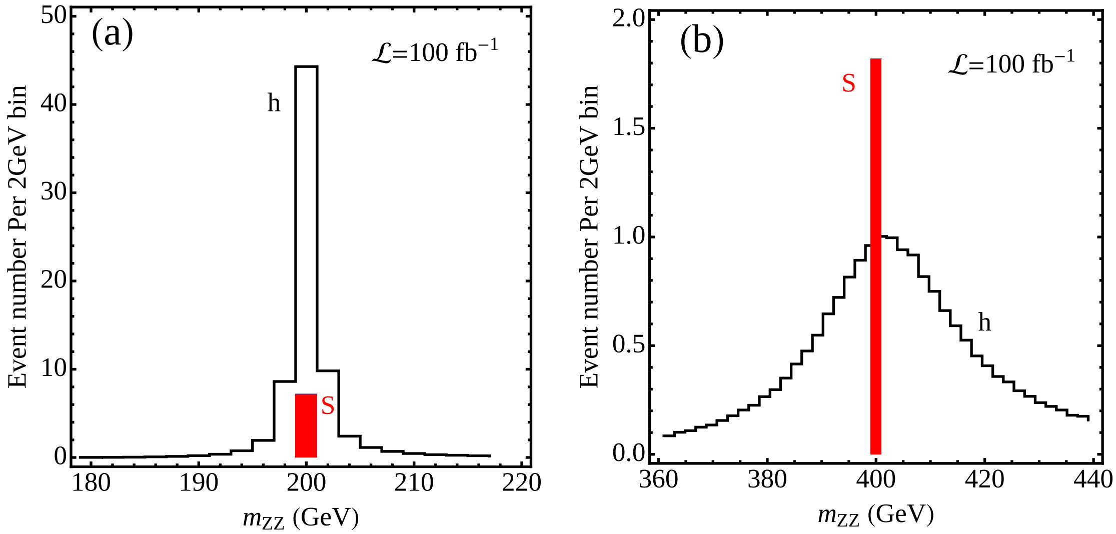

FIG. 3: *The ZZ invariant mass distribution for a SM Higgs boson and a scalar S decaying through loop-induced effects, using a 2 GeV bin size. The narrow width of S is below the detector resolution, resulting in a concentration of all events in just one bin. Note that for a sufficiently small bin size one would resolve the peak, albeit with a form which is dominated by the detector resolution (a Gaussian, if the usual assumptions of detector smearing are made). In the plot we assume the event rate of $gg \to S \to ZZ \to 4\ell$ is only 10% of rate for the SM Higgs boson.*

follows that

$$\frac{\Gamma_{\rm tot}(S)}{\Gamma_{\rm tot}(h)} = \frac{\Gamma(S \to ZZ)}{{\rm Br}(S \to ZZ)} \times \frac{{\rm Br}(h \to ZZ)}{\Gamma(h \to ZZ)} \sim 10^{-3} \,. \tag{45}$$

In other words, we would observe an extremely narrow peak in the ZZ invariant mass spectrum if the scalar S only decays via c_2 and c_3. In fact, the peak is so narrow that the width is completely below the detector resolution. In this study we use a 2 GeV bin size which is comparable to the energy resolution of the detector. On the other hand, a scalar participating in the EWSB like the Higgs boson would have a width above the detector resolution at the LHC except when its mass is below 200 GeV. Therefore, the Breit-Wigner line shape of a scalar resonance in the ZZ invariant mass spectrum is a strong indicator on the Higgs nature (or the lack thereof) of the scalar.

In Fig. 2 we show the total decay widths of a SM Higgs boson and a scalar S decaying through loop-induced operators, and compare with the detector resolution at the LHC using

the following lepton energy smearing:

$$\frac{\delta E}{E} = \frac{a}{\sqrt{E/\mathrm{GeV}}} \oplus b \ , \tag{46}$$

where $a = 13.4\%$, $b = 2\%$, and $\oplus$ denotes a sum in quadrature [22]. We see while the width of the SM Higgs could be resolved above a 200 GeV mass, the small width of the S is completely buried in the detector resolution. The narrow width of S implies, in the invariant mass distribution of the two Z bosons, all the events would be concentrated in just one bin, resulting in a spectacular resonance peak even if the event rate is smaller than that of a SM Higgs. However, for a sufficiently small bin size, one would resolve the peak, albeit with a form which is dominated by the detector resolution (a Gaussian, if the usual assumptions of detector smearing are made). In Fig. 3 we simulate the ZZ invariant mass distribution for a SM Higgs and the S scalar using a 2 GeV bin size. To be conservative, in the plot we assume the event rate $B\sigma(gg \to S \to ZZ \to 4\ell)$ is only 10% of the SM Higgs. It is then clear that the total width measurement would allow for a distinction between the Higgs and a scalar S which decays only at the loop-level, except when the Higgs has a mass below 200 GeV and its width is comparable to the detector resolution.

B. Angular Distributions in ϕ

In the following we consider the dependence on the azimuthal angle between the two Z decay planes in the normalized differential rate in Eq. (15), by turning on one operator at a time. By considering the normalized rate, the dependence on the magnitude of the coefficients c_i in Eq. (6) drops out and the angular dependence is largely determined by kinematics. It is worth mentioning that in Eq. (15) the $\cos(\phi+\delta)$ term is highly suppressed due to the approximate symmetry $g_L^2 \approx g_R^2$ in the leptonic decays, so only the $\cos(2\phi+2\delta)$ term and the constant term will contribute. As can be seen from Eq. (17), for a Higgs-like scalar, $c_1 \neq 0$, the constant term becomes more dominant as the mass gets larger. On the other hand, for $c_2 \neq 0$ the $\cos 2\phi$ terms is more important for a heavy scalar.

In Fig. 4 we simulate the azimuthal angular dependence in the normalized decay distribution for two different scalar masses: 200 and 400 GeV. To facilitate future experimental analysis, we provide in Appendix B Lorentz-invariant expressions for various angular variables in the decay into $ZZ \to 4\ell$, including the azimuthal angle ϕ. We select the events by

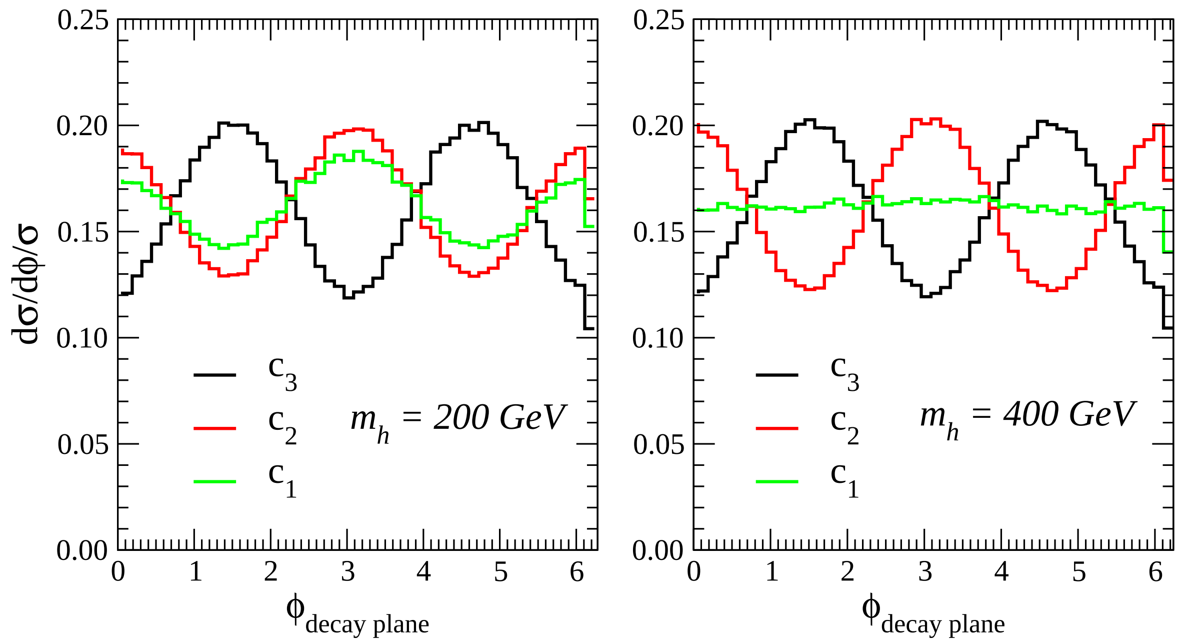

FIG. 4: *The normalized azimuthal angular distributions for 200 and 400 GeV scalar masses, turning on one operator at a time.*

requiring the following cuts on the lepton transverse momentum p_T and rapidity η:

$$p_T \geq 15 \;\; \text{GeV} \qquad \text{and} \qquad |\eta| \leq 2.4 \; . \tag{47}$$

In the simulation we assume the SM background coming from $q\bar{q} \to ZZ \to 4\ell$ has been reduced in the data sample using existing procedures for searching for a SM Higgs boson [2]. The angular dependence of the SM background and its interplay in the Higgs search was studied in Ref. [23]. In the case of loop-induced decays, one can take advantage of the extremely narrow width and impose stringent cuts on the ZZ invariant mass to reduce the backgrounds. Therefore we do not include backgrounds in the plot.

From Fig. 4 we see that the CP-odd case can be distinguished from the CP-even case in the angular distribution, which has been discussed in Refs. [11, 24]. The comparison of tree-level versus loop-induced operators in the CP-even case, however, does not seem to exist in the literature, to the best of our knowledge. We see that, even though $c_1 \neq 0$ and $c_2 \neq 0$ have the same phase in the angular distribution, the magnitudes are different even for a low mass of 200 GeV. Recall that this is also the mass range where the width measurement could be biased by the detector resolution. So one could use the angular distribution as an extra handle to distinguish a Higgs boson from a non-Higgs-like scalar.

V. CONCLUSION

In this work we considered the most general on-shell couplings of a scalar with two Z bosons. In the SM the decay of the Higgs boson into ZZ final states is the gold-plated mode for discovery due to excellent energy resolution for charged leptons. However, in order to verify the Higgs mechanism as the origin of mass for the electroweak gauge bosons, it is necessary to measure the coupling between the scalar and the gauge bosons. By using an operator analysis for the most general couplings, we point out that dimension-five operators responsible for the anomalous Higgs couplings are generated only at the loop-level, while the Higgs mechanism would lead to a dimension-three operator at the tree-level.

Using the method of helicity amplitudes, we computed the differential decay distribution of a scalar decaying into $ZZ \to 4\ell$. Our formulas are simpler than and agree with previous calculations. Furthermore, our results make clear the advantage of using the azimuthal angle between the two decay planes of the Z bosons in discerning effects between CP-odd and CP-even operators.

If the scalar is produced in the gluon fusion channel, which gives the largest production cross-section for the Higgs boson at the LHC, a decay channel into two gluons must also exist. Then in order for the event $gg \to S \to ZZ$ to be observable at the LHC in the early running, the partial decay width should be comparable to the partial width into two gluons. We investigated new physics scenarios giving rise to such a possibility by considering fermion-loop and W'-loop induced couplings of a scalar with ZZ bosons.

One important implication of loop-induced operators is that the total width of a non-Higgs-like scalar, if its decay were discovered at the LHC early on, should be order-of-magnitude smaller than that of a Higgs-like scalar, which decays through tree-level processes. Again this is a corollary of requiring a sizable branching ratio into ZZ final states from loop-induced effects. Therefore measurements of the total width of a scalar resonance in final states with two Z bosons is a strong indicator on the Higgs nature of the resonance, except when the scalar mass is below 200 GeV and the SM Higgs width is comparable to detector resolution. In this regard, azimuthal angular distribution could provide an extra handle in determining not only the CP property of the scalar but also whether the decay is loop-induced. Only when the scalar coupling with the Z bosons is verified to be the one as predicted by the Higgs mechanism, can one gain confidence in the Higgs mechanism as the

origin of electroweak symmetry breaking as well as the discovery of a Higgs boson.

VI. ACKNOWLEDGEMENTS

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Appendix A: ZZ Self Energy From the Heavy Gauge Boson Loop

In this Appendix, we compute the one-loop corrections to the Z self energies which are needed to construct the SZZ effective coupling using the low-energy Higgs theorem. For concreteness, we consider a simple gauge extension of the Standard Model (SM) which is based on the gauge group $SU(2)_1 \times SU(2)_2 \times U(1)_Y$. We use two link fields Σ_1 and Σ_2,

$$\Sigma_1 = \frac{S}{f_1} e^{i\pi_1^a \sigma^a / f_1} \,, \qquad \langle S \rangle = f_1 \,, \tag{48}$$

$$\Sigma_2 = e^{i\pi_2^a \sigma^a / f_2} \,, \tag{49}$$

which transform as bidoublets under $SU(2)_1 \times SU(2)_2$ and $SU(2)_1 \times U(1)_Y$, respectively. The remaining unbroken gauge group is identified with $U(1)_{em}$, whose generator is $Q = T_3^{(1)} + T_3^{(2)} + Y/2$. Notice that the gauge sector of this model is identical to the so-called three-site Higgsless model studied in Refs. [19–21], except that we are allowing for a scalar degree of freedom in the radial excitation of Σ_1. More importantly, we are interested in the limit $\epsilon \equiv (f_2/f_1)^2 \ll 1$, which is also different from the three-site Higgsless model.

The covariant derivatives are written as

$$D_\mu \Sigma_1 = \partial_\mu \Sigma_1 - i g_1 \frac{\sigma^a}{2} W_{1\mu}^a \Sigma_1 + i \Sigma_1 g_2 \frac{\sigma^a}{2} W_{2\mu}^a \,, \tag{50}$$

$$D_\mu \Sigma_2 = \partial_\mu \Sigma_2 - i g_2 \frac{\sigma^a}{2} W_{2\mu}^a \Sigma_2 + i \Sigma_2 g' \frac{\sigma^3}{2} B_\mu, \tag{51}$$

where $W^a_{i\mu}$ and g_i are the gauge fields and coupling strengths belonging to $SU(2)_i$, $i = 1, 2$, respectively. Similarly B_μ and g' correspond to the gauge field and coupling of the $U(1)_Y$. The gauge bosons in the model obtain masses through the kinetic terms

$$\frac{f_1^2}{2}(D_\mu \Sigma_1)^\dagger (D^\mu \Sigma_1) + \frac{f_2^2}{2}(D_\mu \Sigma_2)^\dagger (D^\mu \Sigma_2), \tag{52}$$

which lead to the the mass matrix for neutral gauge bosons (W_1^3, W_2^3, B):

$$\frac{1}{2}\begin{pmatrix} g_1^2 f_1^2 & -g_1 g_2 f_1^2 & 0 \\ -g_1 g_2 f_1^2 & g_2^2(f_2^2 + f_1^2) & -g' g_2 f_2^2 \\ 0 & -g' g_2 f_2^2 & g'^2 f_2^2 \end{pmatrix} . \tag{53}$$

This matrix can be diagonalized by means of an orthogonal matrix which we shall call $\mathbf{R}$:

$$\begin{pmatrix} W^3_{1\mu} \\ W^3_{2\mu} \\ B_\mu \end{pmatrix} = \mathbf{R}^\dagger \begin{pmatrix} A_\mu \\ Z_\mu \\ Z'_\mu \end{pmatrix} , \tag{54}$$

where the mass eigenstates are denoted by A, Z, and Z'. The eigenstate A is massless and identified as the photon. The couplings of our theory are related to the electric charge by

$$g_1 = \frac{e}{\cos\phi \sin\theta_W} , \quad g_2 = \frac{e}{\sin\phi \sin\theta_W} , \quad g' = \frac{e}{\cos\theta_W} \tag{55}$$

where θ_W is the weak mixing angle (in the limit $\epsilon \to 0$) and ϕ is an additional mixing angle. The other two eigenmasses are

$$m_Z^2 = \frac{1}{2} f_2^2 (g^2 + g'^2) \left[1 - \epsilon f_1^2 \frac{g_2^4}{(g_1^2 + g_2^2)}\right] , \tag{56}$$

$$m_{Z'}^2 = \frac{1}{2} f_1^2 (g_1^2 + g_2^2) \left[1 - \epsilon f_1^2 \frac{g_2^4}{(g_1^2 + g_2^2)}\right] , \tag{57}$$

where we have dropped $\mathcal{O}(\epsilon^2)$ terms and

$$\frac{1}{g^2} \equiv \frac{1}{g_1^2} + \frac{1}{g_2^2} . \tag{58}$$

Clearly, Z is identified with the SM Z boson while Z' is referred to as the heavy Z boson. For small ϵ, the mixing matrix $\mathbf{R}$ has the following approximate form:

$$\mathbf{R} = \begin{pmatrix} \cos\phi \sin\theta_W & \sin\phi \sin\theta_W & \cos\theta_W \\ \cos\phi \cos\theta_W + \epsilon \frac{\cos^3\phi \sin^2\phi}{\cos\theta_W} & \sin\phi \cos\theta_W - \epsilon \frac{\sin\phi \cos^4\phi}{\cos\theta_W} & -\sin\theta_W \\ -\sin\phi + \epsilon \sin\phi \cos^4\phi & \cos\phi + \epsilon \sin^2\phi \cos^3\phi & -\epsilon \tan\theta_W \sin\phi \cos^3\phi \end{pmatrix} , \tag{59}$$

from which it is simple to verify that tree-level couplings between the scalar S and the Z boson start only at order ϵ^2. In other words, in this model $c_1 = 0$ at tree-level if we only keep terms up to $\mathcal{O}(\epsilon)$. The charged gauge boson sector can be worked out in a similar way, where the light mass eigenstate is identified with the SM W boson and the heavy eigenstate is denoted by W'. The couplings between S and the W' and Z' bosons have the form as predicted by the Higgs mechanism:

$$m_{V'}^2 \left(1 + \frac{S}{f_1}\right)^2 V'_\mu V'^\mu \ , \quad V = W, Z \ , \tag{60}$$

which is valid at leading order in ϵ.

We would like to compute the one-loop correction to the Z self energy arising from loops of W' gauge bosons. Unfortunately, much like the analogous corrections in the SM, the corrections to the two-point functions depend non-trivially on the particular R_ξ gauge used to define the W' propagator [25]. However, by extracting R_ξ gauge-dependent pieces from other one-loop corrections (i.e., vertex and box corrections) and summing these with those from the two-point function one can obtain an expression which is independent of the particular gauge chosen to do the calculation. This method, which is known as the Pinch Technique (PT), has been applied to the SM to obtain gauge-independent expressions for the gauge boson self-energies [26]. More recently, though, it has been extended to models with extended gauge sectors such as the model considered here [27–29]. In this work, we will directly apply the results from the above references by taking the limit of our interest, $\epsilon = (f_2/f_1)^2 \ll 1$. We refer interested readers to Ref. [27] for details and only make the following two comments. First, our results are obtained by taking the so-called "ideal localization" limit for the de-localized fermion introduced in Ref. [20].[4] Such a limit has the advantage of reducing the tree-level S parameter in the model. However, the main reason in our case is to decouple the de-localized fermion from the W' boson, so as to remove the extra pinch contribution to the two-point function that is unnecessary for maintaining the gauge invariance. Second, even though we are allowing for a scalar degree of freedom in the radial excitation of Σ_1, which is absent in the three-site Higgsless model, the computation in Ref. [27] still carries through because S has no couplings to the Z boson in the order we

[4] In our model ideal localization is achieved by choosing the delocalization parameter x_1, which is defined in Ref. [20], to be $\sin^2\phi/(\sin^2\phi - \cos^2\phi)$.

are working. Therefore the Z self energy in the non-linear sigma model (Higgsless model) is the same as in the (partially) linear sigma model we consider.

The one-loop expression for the Z self-energy computed using the PT are then given by (up to $\mathcal{O}(\epsilon^2)$):

$$\Pi_{ZZ}^{(W')}(p^2) = \frac{\alpha_{em}}{4\pi s_w^2 c_w^2}\left[-\frac{3}{2}\epsilon^2\frac{m_{W'}^4}{m_W^2}\cos^6\phi\sin^6\phi \right.$$
$$\left. + p^2\left(7c_w^4 - 14\,\epsilon\,\cos^2\phi\cos 2\phi\, c_w^2\right)\right]\log\frac{\Lambda^2}{m_{W'}^2}\,, \tag{61}$$

where Λ is the cutoff of our effective theory. Notice that formally $(m_{W'}/m_W)^2 \sim 1/\epsilon$ so the longitudinal piece is considered $m_{W'}^2 \times \mathcal{O}(\epsilon)$, while the leading term in the transverse component has a large coefficient "7," the same as in the SM W contribution to the photon self energy, which is to be expected.

Appendix B: A Lorentz-Invariant Construction of ϕ

In this Appendix we provide a Lorentz-invariant expression for the azimuthal angle ϕ between the two decay planes of the ZZ pair, so as to facilitate the analysis of angular distribution in ϕ. Let p_1 and p_2 be the momenta of the lepton pair coming from one Z, and p_3 and p_4 be the momenta of the lepton pair from the other Z. The parent momentum is $P = p_1 + p_2 + p_3 + p_4$, which satisfies the on-shell dispersion relation $P^2 = M^2$. We follow the notation in Ref. [4], $p_1 = \ell_1$, $p_2 = \bar{\ell}_1$, $p_3 = \ell_2$, $p_4 = \bar{\ell}_2$. (See also Fig. 1.)

In the rest frame of P, our azimuthal angle ϕ is given by

$$\frac{\mathbf{p}_1 \times \mathbf{p_2}}{|\mathbf{p}_1||\mathbf{p}_2|\sin\bar{\theta}_{12}} \cdot \frac{\mathbf{p}_3 \times \mathbf{p_4}}{|\mathbf{p}_3||\mathbf{p}_4|\sin\bar{\theta}_{34}} = -\cos\phi\ , \tag{62}$$

where the triple products in the numerator can be written in a Lorentz-invariant fashion:

$$(\mathbf{p}_1 \times \mathbf{p_2})^i = \frac{1}{M}\epsilon^{\mu\nu i\rho}p_{1\mu}p_{2\nu}P_\rho \equiv \frac{1}{M}\epsilon^{p_1 p_2 i P}\ , \qquad (\mathbf{p}_3 \times \mathbf{p_4})^i = \frac{1}{M}\epsilon^{p_3 p_4 i P}\ . \tag{63}$$

Note that we define $\epsilon^{1230} = 1 = \epsilon_{0123} = -\epsilon^{0123}$. Then it follows

$$(\mathbf{p}_1 \times \mathbf{p_2})\cdot(\mathbf{p}_3 \times \mathbf{p_4}) = -g_{\mu\nu}\frac{\epsilon^{p_1p_2\mu P}\epsilon^{p_3p_4\nu P}}{M^2} = \frac{1}{M^2}\begin{vmatrix} p_1\cdot p_3 & p_1\cdot p_4 & p_1\cdot P \\ p_2\cdot p_3 & p_2\cdot p_4 & p_2\cdot P \\ P\cdot p_3 & P\cdot p_4 & M^2 \end{vmatrix}\ . \tag{64}$$

To arrive at a covariant expression for Eq. (62), we need to cast the denominator in the covariant form as well:

$$|\mathbf{p}_1| = \frac{1}{M}\, p_1 \cdot P \ , \quad \cos\bar{\theta}_{12} = 1 - \frac{m_{12}^2}{2|\mathbf{p}_1||\mathbf{p}_2|} \ , \tag{65}$$

where $m_{ij}^2 \equiv (p_i + p_j)^2$, and similarly for $|\mathbf{p}_2|$,$|\mathbf{p}_3|$, and $|\mathbf{p}_4|$. In the end we have

$$\cos\phi = -\frac{M^2 \begin{vmatrix} p_1\cdot p_3 & p_1\cdot p_4 & p_1\cdot P \\ p_2\cdot p_3 & p_2\cdot p_4 & p_2\cdot P \\ P\cdot p_3 & P\cdot p_4 & M^2 \end{vmatrix}}{(p_1\cdot P)(p_2\cdot P)(p_3\cdot P)(p_4\cdot P)\sqrt{1-\left(1-\frac{M^2 m_{12}^2}{2p_1\cdot P\, p_2\cdot P}\right)^2}\sqrt{1-\left(1-\frac{M^2 m_{34}^2}{2p_3\cdot P\, p_4\cdot P}\right)^2}} \ . \tag{66}$$

On the other hand, $\sin\phi$ can be evaluated by the following relation,

$$\sin\phi = -\frac{1}{M}\frac{\epsilon^{p_1p_2p_3p_4}|\mathbf{p}_1+\mathbf{p}_2|}{|\mathbf{p}_1|\ |\mathbf{p}_2|\ |\mathbf{p}_3|\ |\mathbf{p}_4|\sin\bar{\theta}_{12}\sin\bar{\theta}_{34}} \ , \tag{67}$$

where

$$\epsilon^{p_1p_2p_3p_4} = \epsilon_{\mu\nu\alpha\beta}\, p_1^\mu\, p_2^\nu\, p_3^\alpha\, p_4^\beta = \begin{vmatrix} E_1 & p_1^x & p_1^y & p_1^z \\ E_2 & p_2^x & p_2^y & p_2^z \\ E_3 & p_3^x & p_3^y & p_3^z \\ E_4 & p_4^x & p_4^y & p_4^z \end{vmatrix} = -M\, \mathbf{p_3}\times\mathbf{p_4}\cdot\mathbf{p}_1 \ . \tag{68}$$

The covariant form is given by

$$\sin\phi = -\frac{1}{2}\frac{M^4\lambda^{\frac{1}{2}}\epsilon^{p_1p_2p_3p_4}}{(p_1\cdot P)(p_2\cdot P)(p_3\cdot P)(p_4\cdot P)\sqrt{1-\left(1-\frac{M^2 m_{12}^2}{2p_1\cdot P\, p_2\cdot P}\right)^2}\sqrt{1-\left(1-\frac{M^2 m_{34}^2}{2p_3\cdot P\, p_4\cdot P}\right)^2}} \ , \tag{69}$$

with $\lambda \equiv 1 + m_{12}^4/M^4 + m_{34}^4/M^4 - 2m_{12}^2/M^2 - 2m_{34}^2/M^2 - 2m_{12}^2 m_{34}^2/M^4$.

We can also determine the polar angle of p_1 in the rest frame of the 12 pair. A simple Lorentz boost gives

$$\bar{E}_1 = \gamma E_1(1+\beta\cos\theta) = \frac{\bar{E}_1+\bar{E}_2}{m_{12}}\frac{m_{12}}{2}(1+\beta\cos\theta_1) \ , \tag{70}$$

which leads to

$$\cos\theta_1 = \frac{\bar{E}_1-\bar{E}_2}{|\mathbf{p}_1+\mathbf{p}_2|} = \frac{2}{M^2\lambda^{\frac{1}{2}}}(p_1\cdot P - p_2\cdot P) \ . \tag{71}$$

For the polar angle of p_3 in the rest frame of the 34 pair, simply replace p_1 and p_2 by p_3 and p_4, respectively, in the above.

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